1. Show that $\dfrac{6-\sqrt{8}}{\sqrt{2}-1}$ can be written in the form $a + b\sqrt{2}$ where a and b are integers.

$$\frac{6-\sqrt{8}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \checkmark$$

$$= \frac{(6-\sqrt{8})(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{6\sqrt{2}+6-\sqrt{8\times2}-\sqrt{8}}{2+\sqrt{2}-\sqrt{2}-1}$$

$$= \frac{3\sqrt{2}+2}{1}$$

$$= 2+3\sqrt{2} \checkmark$$

Numerator

$6\sqrt{2}+6-\sqrt{8\times2}-\sqrt{8}$

$6\sqrt{2}+6-\sqrt{16}-\sqrt{8}$

$6\sqrt{2}+6-4-\sqrt{8}$

$6\sqrt{2}+2-\sqrt{8}$

$6\sqrt{2}+2-\sqrt{2\times2\times2}$

$6\sqrt{2}+2-\sqrt{2}\times\sqrt{2}\times\sqrt{2}$

$6\sqrt{2}+2-3\sqrt{2} \checkmark$

$3\sqrt{2}+2$

Denominator

$2+\sqrt{2}-\sqrt{2}-1$

$2-1$

$=1$

(Total for Question is 3 marks)

2. $\sqrt{5}(\sqrt{8} + \sqrt{18})$ can be written in the form $a\sqrt{10}$ where a is an integer.

Find the value of a.

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ (Law of Surds)

$$\sqrt{5}(\sqrt{8} + \sqrt{18}) = \sqrt{5}\sqrt{8} + \sqrt{5}\sqrt{18}$$

Expanding out expression

$$= \sqrt{40} + \sqrt{90} \quad (1)$$

$$= \sqrt{4}\sqrt{10} + \sqrt{9}\sqrt{10}$$

Simplification

looking for square numbers which are factors of 40 and 90

$\frac{40}{10} = 4$

as the answer will need to be in the form $a\sqrt{10}$

$$= 2\sqrt{10} + 3\sqrt{10} \quad (1)$$

$$= 5\sqrt{10}$$

$a =$ 5 (1)

(Total for Question is 3 marks)

3. Martin did this question.

Rationalise the denominator of $\dfrac{14}{2+\sqrt{3}}$

Here is how he answered the question.

$$\frac{14}{2+\sqrt{3}} = \frac{14\times(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{28-14\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}+3}$$

$$= \frac{28-14\sqrt{3}}{7}$$

$$= 4-2\sqrt{3}$$

$(2+\sqrt{3})(2-\sqrt{3})$

$= (2\times 2) + (2\times\sqrt{3}) + (2\times -\sqrt{3}) + (\sqrt{3}\times -\sqrt{3})$

$= 4 + 2\sqrt{3} - 2\sqrt{3} - 3$

$\sqrt{3}\times -\sqrt{3} = -\sqrt{9} = -3$

Martin's answer is wrong.

(a) Find Martin's mistake.

$\sqrt{3}\times-\sqrt{3} = -3$, not 3

(1)

Sian did this question.

Rationalise the denominator of $\dfrac{5}{\sqrt{12}}$

Here is how she answered the question.

$$\frac{5}{\sqrt{12}} = \frac{5\sqrt{12}}{\sqrt{12}\times\sqrt{12}}$$

$$= \frac{5\times 3\sqrt{2}}{12}$$

$$= \frac{5\sqrt{2}}{4}$$

$\sqrt{12} = \sqrt{4}\sqrt{3}$

$= 2\sqrt{3}$

Sian's answer is wrong.

(b) Find Sian's mistake.

$\sqrt{12} = 2\sqrt{3}$, not $3\sqrt{2}$

(1)

(Total for Question is 2 marks)

4. Show that $\dfrac{(\sqrt{18}+\sqrt{2})^2}{\sqrt{8}-2}$ can be written in the form $a(b+\sqrt{2})$ where a and b are integers.

$$\frac{(\sqrt{18}+\sqrt{2})^2}{\sqrt{8}-2} \times \frac{\sqrt{8}+2}{\sqrt{8}+2}$$

$$= \frac{(\sqrt{18}+\sqrt{2})^2(\sqrt{8}+2)}{(\sqrt{8}-2)(\sqrt{8}+2)}$$

$$= \frac{(4\sqrt{2})^2(\sqrt{8}+2)}{8+2\sqrt{8}-2\sqrt{8}-4}$$

$$= \frac{32(\sqrt{8}+2)}{4}$$

$$= 8(\sqrt{8}+2)$$

$$= 8(2+2\sqrt{2})$$

$$= 16(1+\sqrt{2})$$

$\sqrt{18} = \sqrt{2} \times \sqrt{9}$

$= 3\sqrt{2}$

$(4\sqrt{2})^2 = 4^2 \times \sqrt{2}^2$

$= 16 \times 2$

$= 32$

$\sqrt{8} = \sqrt{4} \times \sqrt{2}$

$= 2\sqrt{2}$

(Total for Question is 3 marks)

5. (a) Express $\sqrt{3} + \sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

$$\sqrt{3} + \sqrt{12}$$
$$= \sqrt{3} + \sqrt{4 \times 3}$$
$$= \sqrt{3} + (\sqrt{4})(\sqrt{3})$$
$$= \sqrt{3} + 2\sqrt{3} \quad ①$$
$$= \underline{\underline{3\sqrt{3}}}$$

① $3\sqrt{3}$

....
(2)

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

$$\left(\frac{1}{\sqrt{3}}\right)^7 = \frac{(1)^7}{(\sqrt{3})^7}$$

$$(\sqrt{3})^7 = (\sqrt{3})^6 \times (\sqrt{3})^1$$
$$= (3^{\frac{1}{2}})^6 \times (\sqrt{3})^1$$
$$= 3^3 \times \sqrt{3} = 27\sqrt{3} \quad ①$$

$$\therefore \frac{(1)^7}{(\sqrt{3})^7} = \frac{1}{27\sqrt{3}} \xrightarrow{\times\sqrt{3}} \frac{\sqrt{3}}{(27\sqrt{3})(\sqrt{3})} = \frac{\sqrt{3}}{27 \times 3} = \boxed{\frac{\sqrt{3}}{81}}$$

①

① $\frac{\sqrt{3}}{81}$

....
(3)

(Total for Question is 5 marks)

6. (a) Rationalise the denominator of $\frac{22}{\sqrt{11}}$

$\sqrt{a} \times \sqrt{a} = a$

Give your answer in its simplest form. (1)

$$\frac{22}{\sqrt{11}} \frac{\times \sqrt{11}}{\times \sqrt{11}} = \frac{22\sqrt{11}}{\sqrt{11} \times \sqrt{11}} = \frac{22\sqrt{11}}{11} \frac{\div 11}{\div 11} = \frac{2\sqrt{11}}{1} = 2\sqrt{11}$$

$2\sqrt{11}$ (1)

(2)

(b) Show that $\frac{\sqrt{3}}{2\sqrt{3}-1}$ can be written in the form $\frac{a+\sqrt{3}}{b}$ where a and b are integers.

$$\frac{\sqrt{3}}{2\sqrt{3}-1} \frac{\times(2\sqrt{3}+1)}{\times(2\sqrt{3}+1)} = \frac{\sqrt{3}(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)} = \frac{6+\sqrt{3}}{12+\cancel{2\sqrt{3}}-\cancel{2\sqrt{3}}-1} = \frac{6+\sqrt{3}}{11}$$

(1) (1) (1)

$a = 6$ $b = 11$

(3)

7. Show that $\dfrac{\sqrt{180} - 2\sqrt{5}}{5\sqrt{5} - 5}$ can be written in the form $a + \dfrac{\sqrt{5}}{b}$ where a and b are integers.

$$\frac{a}{b-c} = \frac{a(b+c)}{(b-c)(b+c)} \qquad b^2 - c^2$$

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{180} = \sqrt{9 \times 20}$$
$$= \sqrt{9} \times \sqrt{20}$$
$$= 3 \times \sqrt{20}$$
$$= 3 \times \sqrt{4 \times 5}$$
$$= 3 \times \sqrt{4} \times \sqrt{5}$$
$$= 3 \times 2 \times \sqrt{5} = 6\sqrt{5} \quad ✓_1$$

$$\frac{6\sqrt{5} - 2\sqrt{5}}{5\sqrt{5} - 5} = \frac{4\sqrt{5}}{5\sqrt{5} - 5}$$

$$= \frac{4\sqrt{5}(5\sqrt{5} + 5)}{(5\sqrt{5} - 5)(5\sqrt{5} + 5)} \quad ✓_2$$

$$= \frac{100 + 20\sqrt{5}}{125 - 25}$$

$$= \frac{100 + 20\sqrt{5}}{100} \quad ✓_3$$

$$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$$

$$= \frac{\cancel{100}^{1}}{\cancel{100}_{1}} + \frac{2\cancel{0}\sqrt{5}}{10\cancel{0}}$$

$$= 1 + \frac{\sqrt{5}}{5} \quad ✓_4$$

$$\therefore a = 1$$
$$b = 5$$

(Total for Question is 4 marks)

8. Show that $\dfrac{8+\sqrt{12}}{5+\sqrt{3}}$ can be written in the form $\dfrac{a+\sqrt{3}}{b}$, where a and b are integers.

Rationalise the denominator using 'Difference of two squares.'

$$\frac{8+\sqrt{12}}{5+\sqrt{3}} \quad \begin{matrix}\times(5-\sqrt{3}) \\ \times(5-\sqrt{3})\end{matrix} \quad (1)$$

$\boxed{\sqrt{a} \times \sqrt{a} = a}$

$$= \frac{(8+\sqrt{12})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} \quad (1)$$

$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$\therefore 5\sqrt{12} = 5 \times 2\sqrt{3} = 10\sqrt{3}$

$$= \frac{40 - 8\sqrt{3} + 5\sqrt{12} - (\sqrt{3})(\sqrt{12})}{25 - 5\sqrt{3} + 5\sqrt{3} - (\sqrt{3})(\sqrt{3})}$$

$$= \frac{40 - 8\sqrt{3} + 10\sqrt{3} - \sqrt{36}}{25 - 3}$$

$$= \frac{40 + 2\sqrt{3} - 6}{22} = \frac{34 + 2\sqrt{3}}{22} \quad (1)$$

(Total for Question is 4 marks)

$$= \boxed{\frac{17+\sqrt{3}}{11}} \quad (1)$$