



$$y = a \cos x^\circ + b$$

When $x = 0$
 $y = 3$

$$3 = a \times \cos 0^\circ + b \quad \cos 0^\circ = 1$$

$$3 = a \times 1 + b$$

$$3 = a + b$$

$$3 = a + 1$$

$$-1 \quad -1$$

$$2 = a$$

When $x = 90$

$$y = 1$$

$$1 = a \times \cos 90^\circ + b$$

$$1 = a \times 0 + b$$

$$1 = b$$

When $x = 45$

$$y = 2 \times \cos 45^\circ + 1$$

$$y = 2 \times \frac{\sqrt{2}}{2} + 1$$

$$y = \sqrt{2} + 1$$

$$\sqrt{2} + 1$$

1. Show that $\frac{6 - \sqrt{8}}{\sqrt{2} - 1}$ can be written in the form $a + b\sqrt{2}$ where a and b are integers.

$$\frac{6 - \sqrt{8}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \frac{(6 - \sqrt{8})(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$$

$$= \frac{6\sqrt{2} + 6 - \sqrt{8} \times 2 - \sqrt{8}}{2 + \sqrt{2} - \sqrt{2} - 1}$$

$$= \frac{3\sqrt{2} + 2}{1}$$

$$= 2 + 3\sqrt{2}$$

Numerator

$$6\sqrt{2} + 6 - \sqrt{8} \times 2 - \sqrt{8}$$

$$6\sqrt{2} + 6 - \sqrt{16} - \sqrt{8}$$

$$6\sqrt{2} + 6 - 4 - \sqrt{8}$$

$$6\sqrt{2} + 2 - \sqrt{8}$$

$$6\sqrt{2} + 2 - \sqrt{2} \times 2 \times 2$$

$$6\sqrt{2} + 2 - \sqrt{2} \times \sqrt{2} \times \sqrt{2}$$

$$6\sqrt{2} + 2 - 3\sqrt{2}$$

$$3\sqrt{2} + 2$$

Denominator

$$2 + \sqrt{2} - \sqrt{2} - 1$$

$$2 - 1$$

$$= 1$$

(Total for Question is 3 marks)

an expression to represent any odd number
 $2n + 1$ ①

$$(2n + 1)^2 = (2n + 1)(2n + 1)$$

odd²

$$= 4n^2 + 2n + 2n + 1$$
 ①

$$= 4n^2 + 4n + 1$$
 ①

$$= 4(n^2 + n) + 1$$

is one more than a multiple of 4
as required

a multiple of 4 + 1

2. $\sqrt{5}(\sqrt{8} + \sqrt{18})$ can be written in the form $a\sqrt{10}$ where a is an integer.

Find the value of a .

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \quad (\text{Law of surds})$$

$$\begin{aligned} \sqrt{5}(\sqrt{8} + \sqrt{18}) &= \sqrt{5}\sqrt{8} + \sqrt{5}\sqrt{18} \\ &= \sqrt{40} + \sqrt{90} \quad \text{①} \\ &= \sqrt{4}\sqrt{10} + \sqrt{9}\sqrt{10} \\ &= 2\sqrt{10} + 3\sqrt{10} \quad \text{①} \\ &= 5\sqrt{10} \end{aligned}$$

Expanding out expression

Looking for square numbers which are factors of 40 and 90
 Simplification
 $\frac{40}{10} = 4$
 as the answer will need to be in the form $a\sqrt{10}$

$$a = 5 \quad \text{①}$$

(Total for Question is 3 marks)

3. Martin did this question.

Rationalise the denominator of $\frac{14}{2 + \sqrt{3}}$

Here is how he answered the question.

$$\begin{aligned}\frac{14}{2 + \sqrt{3}} &= \frac{14 \times (2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} \\ &= \frac{28 - 14\sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} + 3} \\ &= \frac{28 - 14\sqrt{3}}{7} \\ &= 4 - 2\sqrt{3}\end{aligned}$$

$$\begin{aligned}&(2 + \sqrt{3})(2 - \sqrt{3}) \\ &= (2 \times 2) + (2 \times \sqrt{3}) + (2 \times -\sqrt{3}) + (\sqrt{3} \times -\sqrt{3}) \\ &= 4 + 2\sqrt{3} - 2\sqrt{3} - 3 \\ &\sqrt{3} \times -\sqrt{3} = -\sqrt{9} = -3\end{aligned}$$

Martin's answer is wrong.

(a) Find Martin's mistake.

$$\sqrt{3} \times -\sqrt{3} = -3, \text{ not } 3$$

(1)

Sian did this question.

Rationalise the denominator of $\frac{5}{\sqrt{12}}$

Here is how she answered the question.

$$\begin{aligned}\frac{5}{\sqrt{12}} &= \frac{5\sqrt{12}}{\sqrt{12} \times \sqrt{12}} \\ &= \frac{5 \times 3\sqrt{2}}{12} \\ &= \frac{5\sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3} \\ &= 2\sqrt{3}\end{aligned}$$

Sian's answer is wrong.

(b) Find Sian's mistake.

$$\sqrt{12} = 2\sqrt{3}, \text{ not } 3\sqrt{2}$$

(1)

(Total for Question 3 is 2 marks)

4. Show that $\frac{(\sqrt{18} + \sqrt{2})^2}{\sqrt{8} - 2}$ can be written in the form $a(b + \sqrt{2})$ where a and b are integers.

$$\begin{aligned} & \frac{(\sqrt{18} + \sqrt{2})^2}{\sqrt{8} - 2} \times \frac{\sqrt{8} + 2}{\sqrt{8} + 2} \quad \checkmark \\ &= \frac{(\sqrt{18} + \sqrt{2})^2 (\sqrt{8} + 2)}{(\sqrt{8} - 2)(\sqrt{8} + 2)} \\ &= \frac{(4\sqrt{2})^2 (\sqrt{8} + 2)}{8 + 2\sqrt{8} - 2\sqrt{8} - 4} \\ &= \frac{32(\sqrt{8} + 2)}{4} \quad \checkmark \\ &= 8(\sqrt{8} + 2) \\ &= 8(2 + 2\sqrt{2}) \\ &= 16(1 + \sqrt{2}) \quad \checkmark \end{aligned}$$

$$\begin{aligned} \sqrt{18} &= \sqrt{2} \times \sqrt{9} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} (4\sqrt{2})^2 &= 4^2 \times \sqrt{2}^2 \\ &= 16 \times 2 \\ &= 32 \end{aligned}$$

$$\begin{aligned} \sqrt{8} &= \sqrt{4} \times \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

(Total for Question is 3 marks)

5. (a) Express $\sqrt{3} + \sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

$$\begin{aligned} & \sqrt{3} + \sqrt{12} \\ &= \sqrt{3} + \sqrt{4 \times 3} \\ &= \sqrt{3} + (\sqrt{4})(\sqrt{3}) \\ &= \sqrt{3} + 2\sqrt{3} \quad \textcircled{1} \\ &= \underline{\underline{3\sqrt{3}}} \end{aligned}$$

$$\begin{aligned} & \textcircled{1} \quad 3\sqrt{3} \\ & \text{-----} \\ & \textcircled{2} \end{aligned}$$

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

$$\begin{aligned} \left(\frac{1}{\sqrt{3}}\right)^7 &= \frac{(1)^7}{(\sqrt{3})^7} \quad \textcircled{1} \\ & \quad \quad \quad \begin{aligned} (\sqrt{3})^7 &= (\sqrt{3})^6 \times (\sqrt{3})^1 \\ &= (3^{\frac{1}{2}})^6 \times (\sqrt{3})^1 \\ &= 3^3 \times \sqrt{3} = 27\sqrt{3} \quad \textcircled{1} \end{aligned} \end{aligned}$$

$$\begin{aligned} \therefore \frac{(1)^7}{(\sqrt{3})^7} &= \frac{1}{27\sqrt{3}} \begin{array}{l} \xrightarrow{\times \sqrt{3}} \\ \xrightarrow{\times \sqrt{3}} \end{array} \frac{\sqrt{3}}{(27\sqrt{3})(\sqrt{3})} = \frac{\sqrt{3}}{27 \times 3} = \boxed{\frac{\sqrt{3}}{81}} \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} & \textcircled{1} \quad \frac{\sqrt{3}}{81} \\ & \text{-----} \\ & \textcircled{3} \end{aligned}$$

(Total for Question is 5 marks)

6. (a) Rationalise the denominator of $\frac{22}{\sqrt{11}}$

$$\sqrt{a} \times \sqrt{a} = a$$

Give your answer in its simplest form. ①

$$\frac{22}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}} = \frac{22\sqrt{11}}{\sqrt{11} \times \sqrt{11}} = \frac{22\sqrt{11}}{11} \div 11 = \frac{2\sqrt{11}}{1} = 2\sqrt{11}$$

$$2\sqrt{11} \quad \text{①}$$

(2)

- (b) Show that $\frac{\sqrt{3}}{2\sqrt{3}-1}$ can be written in the form $\frac{a+\sqrt{3}}{b}$ where a and b are integers.

$$\frac{\sqrt{3}}{2\sqrt{3}-1} \times \frac{(2\sqrt{3}+1)}{(2\sqrt{3}+1)} = \frac{\sqrt{3}(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)} = \frac{6+\sqrt{3}}{12+2\sqrt{3}-2\sqrt{3}-1} = \frac{6+\sqrt{3}}{11}$$

$$a = 6 \quad b = 11$$

(3)

7. Show that $\frac{\sqrt{180} - 2\sqrt{5}}{5\sqrt{5} - 5}$ can be written in the form $a + \frac{\sqrt{5}}{b}$ where a and b are integers.

$$\begin{aligned}\sqrt{180} &= \sqrt{9 \times 20} \\ &= \sqrt{9} \times \sqrt{20} \\ &= 3 \times \sqrt{20} \\ &= 3 \times \sqrt{4 \times 5} \\ &= 3 \times \sqrt{4} \times \sqrt{5} \\ &= 3 \times 2 \times \sqrt{5} = 6\sqrt{5} \quad \checkmark_1\end{aligned}$$

$$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$$

$$\begin{aligned}\therefore a &= 1 \\ b &= 5\end{aligned}$$

$$\frac{a}{b-c} = \frac{a(b+c)}{(b-c)(b+c)}$$

$$b^2 - c^2$$

$$\frac{6\sqrt{5} - 2\sqrt{5}}{5\sqrt{5} - 5} = \frac{4\sqrt{5}}{5\sqrt{5} - 5}$$

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$= \frac{4\sqrt{5} (5\sqrt{5} + 5)}{(5\sqrt{5} - 5)(5\sqrt{5} + 5)} \quad \checkmark_2$$

$$= \frac{100 + 20\sqrt{5}}{125 - 25}$$

$$= \frac{100 + 20\sqrt{5}}{100} \quad \checkmark_3$$

$$= \frac{100}{100} + \frac{20\sqrt{5}}{100}$$

$$= 1 + \frac{\sqrt{5}}{5} \quad \checkmark_4$$

(Total for Question is 4 marks)

8. Show that $\frac{8 + \sqrt{12}}{5 + \sqrt{3}}$ can be written in the form $\frac{a + \sqrt{3}}{b}$, where a and b are integers.

Rationalise the denominator using 'Difference of two squares.'

$$\frac{8 + \sqrt{12}}{5 + \sqrt{3}} \quad \begin{array}{l} \times (5 - \sqrt{3}) \\ \times (5 - \sqrt{3}) \end{array}$$

①

$$\boxed{\sqrt{a} \times \sqrt{a} = a}$$

$$= \frac{(8 + \sqrt{12})(5 - \sqrt{3})}{(5 + \sqrt{3})(5 - \sqrt{3})}$$

①

$$= \frac{40 - 8\sqrt{3} + 5\sqrt{12} - (\sqrt{3})(\sqrt{12})}{25 - 5\sqrt{3} + 5\sqrt{3} - (\sqrt{3})(\sqrt{3})}$$

$$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \sqrt{3} = 2\sqrt{3}$$

$$\therefore 5\sqrt{12} = 5 \times 2\sqrt{3} = 10\sqrt{3}$$

$$= \frac{40 - 8\sqrt{3} + 10\sqrt{3} - \sqrt{36}}{25 - 3}$$

$$= \frac{40 + 2\sqrt{3} - 6}{22} = \frac{34 + 2\sqrt{3}}{22}$$

①

(Total for Question is 4 marks)

$$= \boxed{\frac{17 + \sqrt{3}}{11}} \quad \text{①}$$